# INDIAN SCHOOL AL WADI AL KABIR

**Worksheet- Vectors and Three-Dimensional Geometry**

**Grade: XII** **18th October 2023**

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| 1. | If a line makes angles $\alpha, \beta, \gamma$ *with the positive direction of coordinate axes, then write* the value of $cos2\alpha + cos2\beta + \cos 2\gamma$ . Ans: -1 |
| 2. | Find the angle between the lines 2x= 3y = -z and 6x = -y = -4z. Ans: $\frac{\pi}{2}$ |
| 3. | Write the coordinates of the point which is the reflection of the point $(\alpha, \beta, \gamma)$ *in the XZ plane.* Ans: $(\alpha, -\beta, \gamma)$ |
| 4. | Find the equation of the line passing through origin and (3, -4 , 2). Ans: $\frac{x}{3} = \frac{y}{-4} = \frac{z}{2} = 1$ |
| 5. | If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ *and* $\vec{b}$ for which $\vec{a} - \sqrt{2}\ \vec{b}$ . Ans: $\frac{\pi}{4}$ |
| 6. | Write the unit vector along the sum of vectors $(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and $\hat{\jmath} + \hat{k}$ on the three axes. Ans: $\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$ |
| 7. | The scalar product of the vectors $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to 1. Find the value of λ. Ans: 1 |
| 8. | If $\vec{a} \neq 0$, $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$ |
| 9. | Find λ when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units. Ans: 5 |
| 10. | Show that the four points A(4, 5, 1), B(0, -1, -1), C(3, 9, 4) and D(-4, 4, 4) are coplanar. |
| 11. | The x coordinate of a point Q on the line joining the points P (2, 2, 1) and R(5, 1, -2) is 4. Find its z coordinate. Ans. -1 |
| 12. | Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ respectively are coplanar. Ans: 9 |
| 13. | Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ , $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ *and* $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ *and* $\vec{b}$ and $\vec{c}.\vec{d} = 15$. Ans: $\vec{d} = \left(\frac{5}{3}\right)(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$. |
| 14. | Two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ *and* $2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using diagonal vectors find the area of the parallelogram. Ans: $2\sqrt{101}$ |
| 15. | Find the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ if $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and hence find a vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ . Ans: $\frac{\pi}{2}$, $2\hat{\imath} - 26\hat{\jmath} - 10\hat{k}$ |

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| 16. | Find the shortest distance between the lines:<br>$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and<br>$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ | Shortest distance $= \left\|\frac{-9}{3\sqrt{2}}\right\| = \frac{3\sqrt{2}}{2}$ |
| 17. | Find the shortest distance between the following lines :<br>$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$<br>$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$ | $\frac{\sqrt{145}}{29}$ |
| 18. | Find the vector and cartesian equations of a line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 1, 1).<br>Ans: $\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7}$, $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 7\hat{k})$ | |
| 19. | Find the value of k if the following lines are perpendicular:<br>$\frac{x+3}{k-5} = \frac{y-1}{1} = \frac{5-z}{-2k-1}$; $\frac{x+2}{-1} = \frac{2-y}{-k} = \frac{z}{5}$. | Ans: k= -1 |
| 20. | Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. | Ans: (1, 0, 7) |
| 21. | Case study based:<br>The equation of motion of a missile are<br>x = 3t, y = -4t, z = t, where the time 't' is given in seconds, and the distance is measured in kilometers.<br>Based on the above answer the following:<br>i) What is the path of the missile?<br>ii) At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?<br>iii) If the position of rocket at a certain instant of time is (5, -8, 10), then what will be the height of the rocket from the ground? (The ground is considered as the xy – plane). |  <br>Ans: i) straight line ii) $\sqrt{650}Km$ iii) 10km |

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